**Inverting Matrices: Determinants and Matrix Multiplication[[1]](#footnote-1)©**

**Determinants**

Square matrices have determinants, which are useful in other matrix operations, especially inversion.

For a second-order square matrix, **A**, , the determinant of **A**,



Consider the following bivariate raw data matrix:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Subject # | 1 | 2 | 3 | 4 | 5 |
| X | 12 | 18 | 32 | 44 | 49 |
| Y | 1 | 3 | 2 | 4 | 5 |

from which the following XY variance-covariance matrix is obtained:

|  |  |  |
| --- | --- | --- |
|  | X | Y |
| X | 256 | 21.5 |
| Y | 21.5 | 2.5 |

 

Think of the variance-covariance matrix as containing information about the two variables – the more variable X and Y are, the more information you have. The total amount of information you have is reduced, however, by any redundancy between X and Y – that is, to the extent that you have covariance between X and Y you have less total information. The determinant of a matrix is sometimes called its **generalized variance**, the total amount of information you have about variance in the scores, after removing the redundancy between the variables – look at how we just computed the determinant – the product of the variances (information) less the product of the covariances (redundancy).

Now think of the information in the X scores as being represented by the width of a rectangle, and the information in the Y scores represented by the height of the rectangle. The area of this rectangle is the total amount of information you have. Since I specified that the shape was rectangular (X and Y are perpendicular to one another), the covariance is zero and the generalized variance is simply the product of the two variances.

Now allow X and Y to be correlated with one another. Geometrically this is reducing the angle between X and Y from 90 degrees to a lesser value. As you reduce the angle the area of the parallelogram is reduced – the total information you have is less than the product of the two variances. When X and Y become perfectly correlated (the angle is reduced to zero) the determinant had been reduced to value zero.

Consider the following bivariate raw data matrix:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Subject # | 1 | 2 | 3 | 4 | 5 |
| X | 10 | 20 | 30 | 40 | 50 |
| Y | 1 | 2 | 3 | 4 | 5 |

from which the following XY variance-covariance matrix is obtained:

|  |  |  |
| --- | --- | --- |
|  | X | Y |
| X | 250 | 25 |
| Y | 25 | 2.5 |

 

**Inverting a Matrix**

Determinants are useful in finding the inverse of a matrix, that is, the matrix that when multiplied by **A** yields the identity matrix. That is, **AA−1** = **A−1A** = **I**

An **identity matrix** has 1’s on its main diagonal, 0’s elsewhere. For example, 

With scalars, multiplication by the inverse yields the scalar identity, 1:  Multiplying by an inverse is equivalent to division: 

The inverse of a 2 \* 2 matrix,  for our example.

Multiplying a scalar by a matrix is easy - simply multiply each matrix element by the scalar, thus,



Now to demonstrate that **A**\***A−1** = **A−1**\***A** = **I**, but multiplying matrices is not so easy.

For a 2 \* 2,





**Third-Order Determinant and Matrix Multiplication**

The determinant of a third-order square matrix,





Matrix multiplication for a 3 x 3



That is,



Isn’t this fun? Aren’t you glad that SAS will do matrix algebra for you? Copy the little program below into the SAS editor and submit it.

**SAS Program**

Proc IML;

reset print;

XY ={

256 21.5,

21.5 2.5};

determinant = det(XY);

inverse = inv(XY);

identity = XY\*inverse;

quit;

Look at the program statements. The “reset print” statement makes SAS display each matrix as it is created. When defining a matrix, one puts brackets about the data points and commas at the end of each row of the matrix.

Look at the output. The first matrix is the variance-covariance matrix from this handout. Next is the determinant of that matrix, followed by the inverted variance-covariance matrix. The last matrix is, within rounding error, an identity matrix, obtained by multiplying the variance-covariance matrix by its inverse.

**SAS Output**

**XY 2 rows 2 cols (numeric)**

**256 21.5**

**21.5 2.5**

**DETERMINANT 1 row 1 col (numeric)**

**177.75**

**INVERSE 2 rows 2 cols (numeric)**

**0.0140647 -0.120956**

**-0.120956 1.440225**

**IDENTITY 2 rows 2 cols (numeric)**

**1 -2.22E-16**

**-2.08E-17 1**

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