TEKS A.2 B

Cab Ride

TAKS Objective 2

The student will demonstrate an understanding of the properties and attributes of functions.

TEKS Math Concepts (A.2)

**Foundations for functions.** The student uses the properties and attributes of functions. The student is expected to:

1. identify [and sketch] the general forms of linear () and quadratic () parent functions;
2. **identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;**
3. interpret situations in terms of given graphs [or create situations that fit given graphs]; and
4. [collect and] organize data, [make and] interpret scatterplots (including recognizing positive, negative, or no correlation for data approximating linear situations), and model predict, and make decisions and critical judgments in problem situations.

**Overview**

Identifying domain and range helps to determine the parameters for the solutions to given situations. Domain and range can also be used to effectively represent the graph of a situation visually either when graphing by hand or when using a graphing calculator.

Instructional Strategies

Discovery & Cooperative Learning/Direct Teaching

Students can utilize real world situations and graphs to explore the domain and range of various functions. Students can also utilize their creative side to come up with sketches of functions with given domain and range restrictions.

Lesson Objective

1. Students will identify the domain and range of a function from the graph of the function.
2. Students will create a possible graph of a function given its domain and range.

**👓**For Teacher’s Eyes Only

Students often have a difficult time remembering how *x*, *y*, domain, and range interrelate with each other. One way to help them remember is the following.

* ***x*** and ***y*** values are in order alphabetically
* **domain** and **range** are in order alphabetically

So the domain is the set of all *x*-values, and the range is the set of all *y*-values.

Continuous graphs are connected. Discrete graphs, on the other hand, are made up of distinct points. The domain and range of continuous graphs can sometimes take the form of an interval of values. Note: The only exception to this is if the graph is a constant function like *y = 5*.

The interval of values can be represented using compound inequalities as well as set builder notation. For example a domain of all *x* values between 4 and 10, inclusive can be written as the compound inequality 4 *x*  10. It can also be written as set-builder notation {*x*| 4 *x*  10}.

The domain and range of discrete graphs take the form of a set of distinct values.

Depending on your school’s curriculum, you may or may not use interval notation to represent domain and range instead of inequalities. Decide ahead of time whether you want students to utilize set builder notation with inequalities or interval notation to describe the domain and range of the function. In interval notation, {*x*| 4 *x*  10} can be written as [4, 10].

Students will utilize a modified version of the 4-corner model to organize the information in the Representing Scenarios Activity Sheet #1.

|  |  |
| --- | --- |
| **Verbal Model of Problem Situation** | **Graph** |
| **Table** | **Did you connect the graph or leave as separate points? Explain why.** |

When using the Representing Scenarios with the students, keep in mind that some students may have difficulty understanding when to connect the points and when to use separate points. For example, in the stock market scenario some students may be tempted to just use separate points instead of connecting the points. This could lead into a discussion of whether the stock still has value between day 1 and day 2, day 2 and day 3, etc.

Some students may also encounter difficulty with the Tub Problem. Helping students to connect this problem back to their experience with the CBR activities in the Walk the Graph Lesson may help them in determining the correct graph. (What happened as you walked away from the CBR? What did the graph look like? What happened as you walked toward the CBR? What did the graph look like then? What happened when you stood still? What did the graph look like?)

Depending on their previous experiences, some students may create a step function for the Bank Teller Problem. This would more likely happen if they labeled their x-axis with the actual times (9:00, 10:00, 11:00 12:00, 1:00, etc.) instead of counts (1st, 2nd, 3rd, 4th, etc.). If any students should happen to create a step function for the graph, then discuss with the class how the step function could also correctly represent the scenario. Using the terminology, step function, may not even be necessary at this point. In any case, this would be a wonderful way to illustrate that there can be more than one “correct” way to examine a situation.

As an extension, when debriefing the Elaborate Activity (Domain and Range Activity), question the students as to which graphs represented functions and which did not. Emphasize that even though some of the graphs are not functions, they are relations – the relation just happens to not be a function since some of the inputs have more than one output. Regardless of whether or not a relation is a function, the relation still has a domain and range.

Misconceptions

🗷 Misconception

* Domain and range are always a set of distinct numbers.
* Domain is the set of *y* values in a relation. (Range is the set of *x* values in a relation.)

🗹 Mathematics Concept

* Domain and range can be a set of distinct numbers if the graph is discrete. Domain and range can also be a continuous interval of values if the graph is continuous.
* The domain of a relation is always the set of first coordinates (or *x*-values) of the ordered pairs, while the range of a relation is always the set of second coordinates (or *y*-values) of the ordered pairs.

🛠Rebuild Concept

* Provide students with examples of both continuous and discrete relations where they have to identify the domain and range.
* Provide students with examples and opportunities to identify and use the domain and range of the graphs of relations.

Student Prior Knowledge

Students have previous experience with representing relationships graphically (TEKS A.1.D). In addition, students have previous experience identifying independent and dependent quantities (TEKS A.1.A).

Materials

* Display Page #1, Cab Ride
* Activity Sheet #1, Representing Scenarios
* Activity Sheet #2, Domain and Range Cards
* Activity Sheet #3, Domain and Range Record Sheet
* Activity Sheet #4, Drawing on Domain and Range
* PowerPoint Presentation – Cab Ride

5 E’s

💣ENGAGE

*The learner is introduced to a new experience and must draw from prior experiences to make sense of the engage activity.*

1. Display Page #1, **Cab Ride**, to the students. Have each student answer the questions on the activity.
2. Once all students have answered the questions, have students share their answers with the class.

👁EXPLORE

*During the explore activity, the student becomes directly involved with a particular phenomena by manipulation of materials that are used to discover the phenomena.*

1. Distribute Activity Sheet #1 **Representing Scenarios** handout to each student. Have students complete the handouts in pairs. Students will be using the 4-corner model for each problem. An alternative approach here would be to students work in groups of 4. Have each student be responsible for writing up the 4-corner model for 1 of the scenarios after the group has discussed each scenario.
2. Once all students have completed the handout, have students share their answers with the class.

🗣EXPLAIN

*The student communicates in verbal and written form about the information derived from the learning experience.*

The domain values of a function represent the input values, or equivalently, the values of the independent variable. Graphically, the domain is represented by the set of *x* values on the *x*-axis that correspond to actual points on the graph of the function.

The range values of a function represent the output values. Graphically, the range is represented by the set of *y* values on the *y*-axis that correspond to actual points on the graph of the function.

Domain and range look different depending on whether the situation is discrete (separate, distinct points) or continuous (a range of values).

Have students identify those situations which were discrete and those that were continuous from the Explore Activity. Ask students to which values make sense and which values do not make sense in the situation. For example, on the School Attendance scenario from the Explore Activity, ask students if it would be possible to have 1.2 six weeks? Or likewise, would it be possible to have 0.35 absences?

In a discrete situation, the domain and range would be written as a list of separate values using set notation { }. Review with students this notation. Students may confuse this notation with ordered pair notation. Ask students to review and identify what the ordered pair notation (5, 2) means. That is 5 represents the *x–*value of the ordered pair and 2 represents the *y*–value of the ordered pair. Similarly, (5, 2) represents a specific location on the coordinate plane corresponding to those specific *x* and *y* values. Illustrate that the set notation, {5, 2}, represents two separate numbers or answers, namely 5 and 2.

For continuous functions, the domain and range could be written as a simple inequality, a compound inequality, or “all real numbers” if there are no restrictions on the values. Students have some experience with compound inequalities from 8th grade math but will need to review what a compound inequality looks like. Question students on what would be an example of a compound inequality. Also, students may or may not have previous experience with the phrase “All real numbers.” Question students on what they think the phrase “all real numbers” means in terms of the previous scenarios they have graphed.

🔍ELABORATE

*During the elaboration phase, students expand their knowledge by making connections about what they have learned and applying this new knowledge to real world situations.*

**Materials**

* Domain and Range Game Cards  
  *Note: These will need to be run off on card stock and cut out prior to playing the game. Be sure to separate the answer cards from the graphs.*

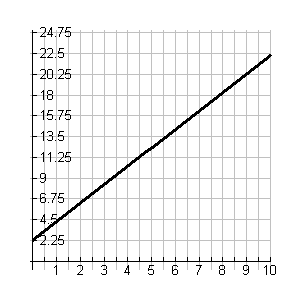
1. Have students partner up to play the **Domain & Range Activity**. Each pair will need one game set, see Activity Sheet #2.
2. Distribute the Activity Sheet #3, **DOMAIN & RANGE RECORD SHEET** to each person. Every student will be responsible for recording the matches they make on this record sheet.
3. The object is to match each graph card with its correct domain and range card.
4. Students need to record the matches they make on their record sheet and check their answers with another group.
5. Once all pairs of students have had a chance to finish up the activity, have the students share with the class which problems they thought were the trickiest and which were the easiest.

🌡EVALUATE

*Evaluation throughout the learning experience is an ongoing process and has a diagnostic function.*

1. Distribute Activity sheet #4, **Drawing on Domain and Range** to each student.
2. Each student needs to complete the activity on their own.
3. Debrief the class.

Display Page #1

Billy needs a ride to the airport so he calls a taxi cab. The taxi cab charges $2.25 to come pick Billy up. The cab company then charges $.20 for each additional 1/10 mile. The graph shows the total cost, *y*, of the taxi cab ride as a function of the number of miles, *x*, traveled.

Looking at the graph, what are the possible values for *x*?

Looking at the graph, what are the possible values for *y*?

Activity Sheet #1

Draw a graph using the scenarios listed below. Then answer the questions that follow.

|  |  |
| --- | --- |
| **Stock Market** – Joe invested in some stock valued at $30. For the next 3 days the value of his stock increased by $5 each day. On the fourth day, his stock had dropped in value $15. It dropped another $5 on the fifth day. On the 6th day, the stock’s value increased $12. Come up with a graph to represent this situation. Create a table of values to help you graph this scenario. | [image] |
| **School Attendance** – Axel tracks the number of absences he has had each six weeks. The numbers of absences are listed below. Come up with a graph to represent this situation.  1st Six Weeks = 2  2nd Six Weeks = 1  3rd Six Weeks = 0  4th Six Weeks = 4  5th Six Weeks = 1  6th Six Weeks = 3  Create a table of values to help you graph this scenario. | [image] |
| **Bank Teller Problem** – A local bank tracks the total number of people that use their drive through each hour. The first count was tallied at 40 customers from 9-10am. The second count was tallied at 28 customers from 10-11am. There were 45 customers from 11-12pm. There were 51 customers from 12-1pm. There were 12 customers from 1-2pm; 31 customers from 2-3pm; 29 customers from 3-4pm; 38 customers from 4-5pm. Finally, there were 54 customers from 5-6pm. Come up with a graph for this situation. Create a table of values to help you graph this scenario. | [image] |
| **Tub Problem** – Madison turns the water faucet on to fill the bathtub for her bath. The water flows into the tub at a constant rate. She turns the water off after 5 minutes when there are 35 gallons of water in the tub. She stays in the tub for 20 minutes. She gets out of the bathtub and pulls the drain. The water drains out of the tub at a constant rate until the tub is empty after 7 minutes. Come up with a graph to represent this situation. Create a table of values to help you graph this scenario. | [image] |

For each of the graphs that you created, identify the *x* values that are represented in the graph.

For each of the graphs that you created, identify the *y* values that are represented in the graph.

Activity Sheet #2

|  |  |  |
| --- | --- | --- |
| *1* |  | *2* |
| This graph has a domain consisting of all real numbers.  This graph has a range consisting of all real numbers.  *Card D* |  | This graph has a domain consisting of all real numbers.  This graph has a range of *y* ≥-1.  *Card H* |
| *3* |  | *4* |
| This graph has a domain consisting of -5≤*x*≤5. This graph has a range of  0≤*y*≤4.  *Card M* |  | This graph has a domain of *x*<0 and *x*>0. This graph has a range of *y*>0.  *Card T* |

|  |  |  |
| --- | --- | --- |
| [image]  *5* |  | [image]  *6* |
| This graph has a domain consisting of all real numbers.  This graph has a range consisting of *y*≥3.  *Card F* |  | This graph has a domain of -3≤*x*≤3.  This graph has a range of -3≤*y* ≤3.  *Card B* |
| *7*  [image] |  | *8* |
| This graph has a domain of -4≤*x*≤5.  This graph has a range of -2≤*y* ≤3.  *Card P* |  | This graph has a domain of  -2≤*x*≤4. This graph has a range of -5≤*y*≤5.  *Card K* |

|  |  |  |
| --- | --- | --- |
| *9* |  | *10* |
| This graph has a domain of 0≤*x*≤4.  This graph has a range of 0≤*y* ≤3.  *Card J* |  | This graph has a domain consisting of all real numbers.  This graph has a range of *y* ≤-2.  *Card S* |
| *11*  [image] |  | *12* |
| This graph has a domain of -1≤*x*≤1.  This graph has a range of 0≤*y* ≤1.  *Card A* |  | This graph has a domain of -1≤*x*≤1. This graph has a range of 1**≤** *y* **≤** 5.  *Card N* |

|  |  |  |
| --- | --- | --- |
| [image]  *13* |  | [image]  *14* |
| This graph has a domain of all real numbers.  This graph has a range of {-3, 3}.  *Card L* |  | This graph has a domain of {-4,-2,-1,0,1,3,4}.  This graph has a range of {-3,-1,2,3,5}.  *Card R* |
| [image]  *15* |  | [image]  *16* |
| This graph has a domain of -3≤*x*≤2.  This graph has a range of 0≤*y* ≤4.  *Card C* |  | This graph has a domain of {-5,-4,-3,-2,-1,0,1,2,3,4,5}. This graph has a range of {-5,-4,-3,-2,-1,0,1,2,3,4,5}.  *Card O* |

|  |  |  |
| --- | --- | --- |
| *17* |  | *18* |
| This graph has a domain of all real numbers.  This graph has a range of *y*≥2.  *Card U* |  | This graph has a domain of all real numbers.  This graph has a range of -2≤*y*≤2.  *Card I* |
| [image]  *19* |  | *20* |
| This graph has a domain of -5≤*x*≤4.  This graph has a range of -2≤*y* ≤3.  *Card G* |  | This graph has a domain of *x*≥0. This graph has a range of all real numbers.  *Card Q* |

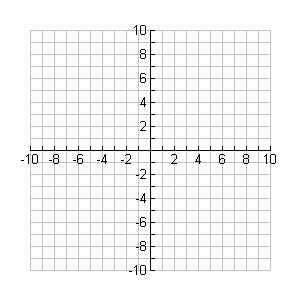
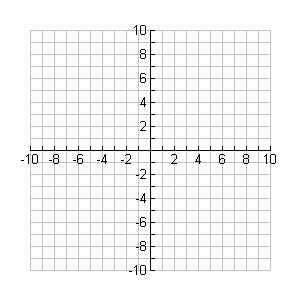
Activity Sheet #3

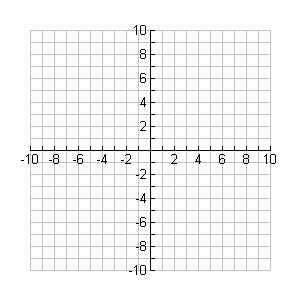
Record the matching pairs of cards that you and your partner find in the chart below.

|  |  |
| --- | --- |
| **Graph Number** | **Matching Card Letter** |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |
| 11 |  |
| 12 |  |
| 13 |  |
| 14 |  |
| 15 |  |
| 16 |  |
| 17 |  |
| 18 |  |
| 19 |  |
| 20 |  |

Activity Sheet #4

Given the following descriptions of the domain and range, sketch a graph that could represent the domain and range.

1. Domain is all real numbers.   
   Range is *y*<0.
2. Domain is -10≤*x*≤6.  
   Range is 0≤*y*≤5.
3. Domain is {-2, 1, 3, 4, 5}.  
   Range is {2, 4}



You are reading several books for a report that you have to write in your English class. You can read 1.5 pages per minute. Complete the table of values below and graph the function represented by the table. Your graph will be continuous.

Identify the domain and range of the function that you graphed.

|  |  |  |
| --- | --- | --- |
| ***# min*** | ***Process*** | ***Pages read*** |
| 0 |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 5 |  |  |
| 10 |  |  |
| *x* |  |  |

